HEAT TRANSFER FROM A CYLINDRICAL, HEAVILY FINNED SURFACE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 6, pp. 783–787, 1965

UDC 536.244

The transfer of heat from a strongly heated, heavily finned surface is examined. The material of the surface and of the fins has high thermal conductivity (copper), while the heat transfer fluid has comparatively low thermal conductivity (oil). Under these conditions, increase in the number of fins and reduction of the distance between them makes possible a high value of the coefficient of heat transfer from the heated surface to the fluid.

We shall examine a cylindrical body (figure) with intense heat transfer from its surface to a fluid flowing in grooves formed in the cylindrical surface in a direction parallel to the axis.

Let us consider the case when the grooves and the fins separating them are small in the tangential direction, and when the temperature may be regarded as constant through the thickness of a fin. Transfer of heat along a fin by conduction is not considered. Neglect of heat transfer through the fin in the longitudinal direction gives a fin temperature in the hottest region that is greater than the actual value. The thermophysical constants of the fluid—specific heat, thermal conductivity, viscosity—are regarded as constant throughout the flow. The coefficient of heat transfer α from the fin surface to the fluid is considered constant over the whole surface of the fin, since the Nusselt number for a plane slot is little affected by the heat transfer conditions (we are concerned with the laminar flow regime, since the groove width $\varkappa$ is small [1]).

The question is to find the temperature field and the coefficient of heat transfer from the heated surface to the cooling fluid. We introduce the coordinate system r, z, as shown in the figure. In this coordinate system we consider the cross section of a fin approximately as part of a sector with center at the point 0_1. The radii of the tip and base of the fin, b and a, are then easily determined in terms of radii R_2 and R_1 (see fig.):

$$b = R_2 - \frac{\varkappa}{2}\,\frac{1}{\sin(\varphi/2)}, \quad a = R_1 - \frac{\varkappa}{2}\,\frac{1}{\sin(\varphi/2)}. \quad (1)$$

With the above assumptions the differential equation of heat transfer for the fin has the form

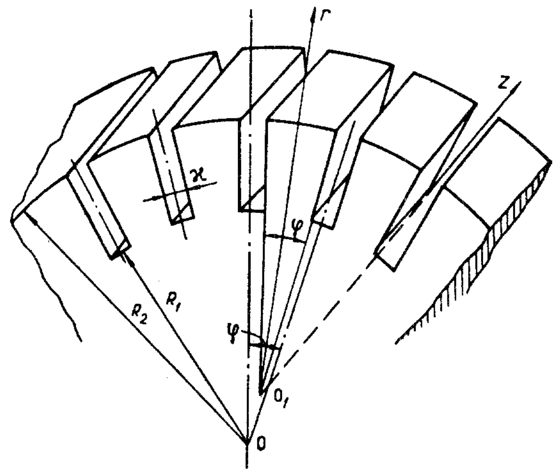
$$\lambda\varphi\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = 2\alpha(t-u). \quad (2)$$

The boundary conditions for (2) are

$$\frac{\partial t}{\partial r} = 0 \text{ when } r = b \quad (3)$$

(no heat flux through the tip of the fin),

$$\frac{\partial t}{\partial r} = -\frac{q}{\lambda} \text{ when } r = a. \quad (4)$$



Sketch of heavily finned cylindrical body.

The differential heat balance equation for the fluid is

$$cv\varkappa\gamma\frac{\partial u}{\partial z} = 2\alpha(t-u). \quad (5)$$

The boundary condition for (5) is

$$u = 0 \text{ when } z = 0 \quad (6)$$

(the temperature of the fluid in the initial section is assumed to be zero).

To solve the system of equations (2) and (5), we perform a Laplace transformation of these equations and their boundary conditions. Here the transformation may be performed both for an infinitely long fin and for a finite one, so that, owing to absence of heat conduction in the longitudinal direction, the fin of finite length may be considered as the first part of an infinite one.

We introduce the notation

$$\bar{t} = \int_0^\infty \exp(pz)\,t(r,z)\,dz; \quad \bar{u} = \int_0^\infty u\exp(pz)\,dz. \quad (7)$$

After transformation we obtain from (2), (3), and (4)

$$2\alpha(\bar{t}-\bar{u}) = \lambda\varphi\frac{d}{dr}\left(r\frac{d\bar{t}}{dr}\right). \quad (8)$$

$$\frac{d\bar{t}}{dr}=0 \text{ when } r=b;\quad \frac{d\bar{t}}{dr}=-\frac{q}{\lambda}\,\frac{1}{p} \text{ when } r=a \tag{9}$$

and from (5) and (6)

$$2\alpha(\bar{t}-\bar{u})=c v\varkappa\gamma p\bar{u}. \tag{10}$$

Eliminating the variable $\bar{t}$ from (8) and (10), we obtain for $\bar{u}$ the equation

$$r\frac{d^2\bar{u}}{dr^2}+\frac{d\bar{u}}{dr}+s\bar{u}=0 \tag{11}$$

with boundary conditions

$$\frac{d\bar{u}}{dr}=0,\quad r=b;$$
$$\frac{d\bar{u}}{dr}=-\frac{q}{\lambda p}\,\frac{1}{1+\frac{cv\varkappa\gamma}{2\alpha}p},\quad r=a, \tag{12}$$

where

$$s=-\frac{2\alpha}{\lambda\varphi}\,\frac{p}{2\alpha/cv\varkappa\gamma+p}. \tag{13}$$

Solving (11) and satisfying boundary conditions (12), we find [2]

$$\bar{u}=\frac{q\varphi}{cv\varkappa\gamma}\,\frac{\sqrt{as}}{p^2}\times$$
$$\times\frac{I_0(2\sqrt{sr})Y_1(2\sqrt{sb})-Y_0(2\sqrt{sr})I_1(2\sqrt{sb})}{I_1(2\sqrt{sb})Y_1(2\sqrt{sa})-I_1(2\sqrt{sa})Y_1(2\sqrt{sb})}. \tag{14}$$

Transforming (14), we find

$$u=\frac{q\varphi}{cv\varkappa\gamma}\,\frac{az}{b-a}+\frac{q}{\lambda}\left\{(b-a)\left(\frac{r}{b}+\ln\frac{b}{r}-1\right)+\right.$$
$$\left.+a\left[\ln\frac{b}{a}-\frac{1}{2}\left(\frac{b}{a}-\frac{a}{b}\right)\right]\right\}\left[\frac{b}{a}+\frac{a}{b}-2\right]^{-1}+$$
$$+\frac{q\varphi}{2\alpha}\pi\sqrt{a}\times$$
$$\times\sum_{n=1}^{\infty}\left\{\exp(p_n z)\left(\frac{2\alpha}{cv\varkappa\gamma}\,\frac{1}{p_n}+1\right)\sqrt{s_n}\,I_1(2\sqrt{s_n a})\,I_1(2\sqrt{s_n b})\times\right.$$
$$\left.\times[I_1^2(2\sqrt{s_n b})-I_1^2(2\sqrt{s_n a})]^{-1}\right\}\times$$
$$\times[Y_0(2\sqrt{s_n r})I_1(2\sqrt{s_n b})-I_0(2\sqrt{s_n r})Y_1(2\sqrt{s_n b})], \tag{15}$$

where s_n are roots of the equation

$$I_1(2\sqrt{sb})Y_1(2\sqrt{sa})-I_1(2\sqrt{sa})Y_1(2\sqrt{sb})=0 \tag{16}$$

(equation (16) has only real roots [3]), and p_n are found in terms of s_n from (13).

It can easily be shown that the series in (15) converges.

We shall find the fin temperature from (5) with the aid of (15):

$$t=\frac{q\varphi}{cv\varkappa\gamma}\,\frac{az}{b-a}+\frac{q\varphi}{2\alpha}\,\frac{a}{b-a}+$$
$$+\frac{qab}{\lambda}\left\{(b-a)\left(\frac{r}{b}+\ln\frac{b}{r}-1\right)+\right.$$
$$\left.+a\left[\ln\frac{b}{a}-\frac{1}{2}\left(\frac{b}{a}-\frac{a}{b}\right)\right]\right\}[(b-a)^2]^{-1}+\frac{q\varphi}{2\alpha}\pi\sqrt{a}\times$$
$$\times\sum_{n=1}^{\infty}\left\{\left(\frac{2\alpha}{cv\varkappa\gamma}\,\frac{1}{p}+2+\frac{cv\varkappa\gamma}{2\alpha}p\right)\sqrt{s_n}I_1(2\sqrt{s_n a})I_1(2\sqrt{s_n b})\times\right.$$
$$\left.\times[I_1^2(2\sqrt{s_n b})-I_1^2(2\sqrt{s_n a})]^{-1}\right\}\times \tag{17}$$
$$\times\exp(p_n z)\left[Y_0(2\sqrt{s_n r})I_1(2\sqrt{s_n b})-I_0(2\sqrt{s_n r})Y_1(2\sqrt{s_n b})\right].$$

We shall also examine the coefficient k, defined as the ratio of the heat flux passing through the surface of radius R_1 to the temperature difference between the base of the fin (cooled surface) and the mean fluid temperature over the fin height u^0

$$k=q/(1+\varkappa/\varphi a)(t_0-u^0), \tag{18}$$

where

$$u^0=\frac{1}{b-a}\int_a^b u\,dr;\quad t_0=t_{r=a}. \tag{19}$$

With the help of (15), (17), and (19) it is easy to compute k at any point on the cooled surface.

Thus the problem has been solved.

It is easy to see that the value of the coefficient k in the initial section k_{max} is greater than in the later sections, i.e., k falls with increasing z. As $z\to\infty$, k tends to some minimum value k_{min}, which is easily found from (18), using (15), (17), and (19), to be

$$k_{\min}=(b-a)\left\{\left(1+\frac{\varkappa}{\varphi a}\right)\left[\frac{\varphi}{2\alpha}a+\right.\right.$$
$$\left.\left.+\frac{ab}{\lambda}\left(\frac{1}{2}\,\frac{a}{b}-\frac{3}{2}+\frac{b}{b-a}\ln\frac{b}{a}\right)\right]\right\}^{-1}. \tag{20}$$

Using (20) we can show how effective the cooling method examined is for a very dense array of fins and very small grooves (channels) between fins. Let $R_2=16.5$ cm, $\varphi=0.00266$, $R_1=15$ cm, $\lambda=3.8$ W/cm$^2\cdot{}^\circ$C, $\varkappa=0.02$ cm, $\alpha=0.25$ W/cm$^2\cdot{}^\circ$C. Then from (20) we have $k_{min}=3.29$ W/cm$^2\cdot{}^\circ$C, i.e., the minimum value of k is quite large. This example corresponds to oil cooling of a heavily fined copper surface. It is not hard to obtain an expression for the maximum value of k in the initial section, if we consider the fluid temperature constant over the entire channel

$$k_{\max}=\frac{\sqrt{2\alpha\lambda/a\varphi}}{1+\varkappa/\varphi a}\,\frac{I_1(\varepsilon b^{1/2})K_1(\varepsilon a^{1/2})-I_1(\varepsilon a^{1/2})K_1(\varepsilon b^{1/2})}{I_0(\varepsilon a^{1/2})K_1(\varepsilon b^{1/2})+K_0(\varepsilon a^{1/2})I_1(\varepsilon b^{1/2})},$$
$$\varepsilon=\sqrt{8\alpha/\lambda\varphi}. \tag{21}$$

In the above example calculation of k without taking account of warming up of the fluid gives $k_{max} = 4.88$ W/cm$^2 \cdot$ °C.

Thus, in practical calculations, it is necessary in a number of cases to use (20) together with (21), particularly since (20) gives a better description of heat transfer at the hottest point of the body—at the fluid outlet.

NOTATION

R_2 and R_1—radii of cylindrical surface at tip and base of fins, respectively; r, z—coordinate axes; φ—angle between planes of adjacent slots; χ—slot width; b and a—coordinates of tip and base of fin; λ—thermal conductivity of fin material; t—fin temperature; u—mean fluid temperature over slot width; α—coefficient of heat transfer from fin surface to fluid; c—specific heat of fluid; v—fluid velocity (since the viscosity is considered constant over the flow, v is also constant); q—density of heat flux through base of fin; γ—specific weight of fluid; $\bar{t}$, $\bar{u}$—Laplace transforms; p—Laplace transform parameter, s—auxiliary quantity determined from equation (13); I, K, J, $\hat{Y}$—Bessel functions; u^0—mean fluid temperature over height of slot.

REFERENCES

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11 February 1965

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